cen58933_ch13.qxd 9/9/2002 9:57 AM Page 680

Two special types of heat exchangers commonly used in practice are *condensers* and *boilers*. One of the fluids in a condenser or a boiler undergoes a phase-change process, and the rate of heat transfer is expressed as

$$\dot{Q} = \dot{m} h_{fg} \tag{13-14}$$

where $\dot{m}$ is the rate of evaporation or condensation of the fluid and h_{fg} is the enthalpy of vaporization of the fluid at the specified temperature or pressure.

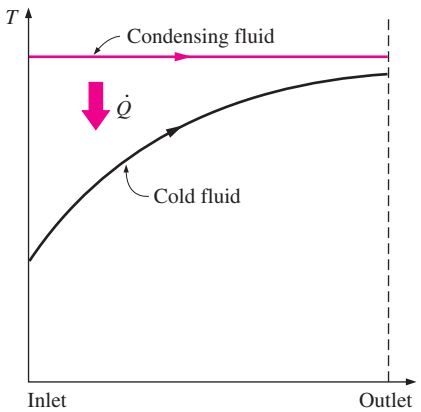
An ordinary fluid absorbs or releases a large amount of heat essentially at constant temperature during a phase-change process, as shown in Figure 13–13. The heat capacity rate of a fluid during a phase-change process must approach infinity since the temperature change is practically zero. That is, $C = \dot{m} C_p \to \infty$ when $\Delta T \to 0$, so that the heat transfer rate $\dot{Q} = \dot{m} C_p \Delta T$ is a finite quantity. Therefore, in heat exchanger analysis, a condensing or boiling fluid is conveniently modeled as a fluid whose heat capacity rate is *infinity*.

The rate of heat transfer in a heat exchanger can also be expressed in an analogous manner to Newton's law of cooling as

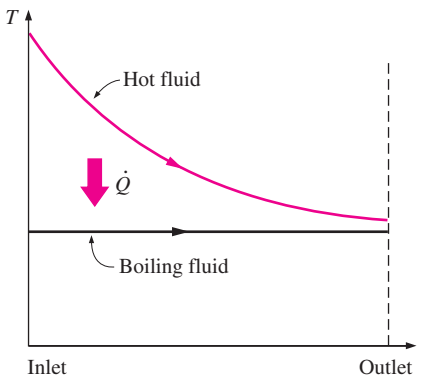
$$\dot{Q} = U A_s \Delta T_m \tag{13-15}$$

where U is the overall heat transfer coefficient, A_s is the heat transfer area, and ΔT_m is an appropriate average temperature difference between the two fluids. Here the surface area A_s can be determined precisely using the dimensions of the heat exchanger. However, the overall heat transfer coefficient U and the temperature difference ΔT between the hot and cold fluids, in general, are not constant and vary along the heat exchanger.

The average value of the overall heat transfer coefficient can be determined as described in the preceding section by using the average convection coefficients for each fluid. It turns out that the appropriate form of the mean temperature difference between the two fluids is *logarithmic* in nature, and its determination is presented in Section 13–4.



(*a*) Condenser ($C_h \to \infty$)



(*b*) Boiler ($C_c \to \infty$)

FIGURE 13–13
Variation of fluid temperatures in a heat exchanger when one of the fluids condenses or boils.

13–4 ▪ THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

Earlier, we mentioned that the temperature difference between the hot and cold fluids varies along the heat exchanger, and it is convenient to have a *mean temperature difference* ΔT_m for use in the relation $\dot{Q} = U A_s \Delta T_m$.

In order to develop a relation for the equivalent average temperature difference between the two fluids, consider the *parallel-flow double-pipe* heat exchanger shown in Figure 13–14. Note that the temperature difference ΔT between the hot and cold fluids is large at the inlet of the heat exchanger but decreases exponentially toward the outlet. As you would expect, the temperature of the hot fluid decreases and the temperature of the cold fluid increases along the heat exchanger, but the temperature of the cold fluid can never exceed that of the hot fluid no matter how long the heat exchanger is.

Assuming the outer surface of the heat exchanger to be well insulated so that any heat transfer occurs between the two fluids, and disregarding any

cen58933_ch13.qxd 9/9/2002 9:57 AM Page 681

changes in kinetic and potential energy, an energy balance on each fluid in a differential section of the heat exchanger can be expressed as

$$\delta\dot{Q} = -\dot{m}_h C_{ph}\, dT_h \tag{13-16}$$

and

$$\delta\dot{Q} = \dot{m}_c C_{pc}\, dT_c \tag{13-17}$$

That is, the rate of heat loss from the hot fluid at any section of a heat exchanger is equal to the rate of heat gain by the cold fluid in that section. The temperature change of the hot fluid is a *negative* quantity, and so a *negative sign* is added to Eq. 13-16 to make the heat transfer rate $\dot{Q}$ a positive quantity. Solving the equations above for dT_h and dT_c gives

$$dT_h = -\frac{\delta\dot{Q}}{\dot{m}_h C_{ph}} \tag{13-18}$$

and

$$dT_c = \frac{\delta\dot{Q}}{\dot{m}_c C_{pc}} \tag{13-19}$$

Taking their difference, we get

$$dT_h - dT_c = d(T_h - T_c) = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{13-20}$$

The rate of heat transfer in the differential section of the heat exchanger can also be expressed as

$$\delta\dot{Q} = U(T_h - T_c)\, dA_s \tag{13-21}$$

Substituting this equation into Eq. 13-20 and rearranging gives

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\, dA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{13-22}$$

Integrating from the inlet of the heat exchanger to its outlet, we obtain

$$\ln\frac{T_{h,\,\text{out}} - T_{c,\,\text{out}}}{T_{h,\,\text{in}} - T_{c,\,\text{in}}} = -UA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{13-23}$$

Finally, solving Eqs. 13-9 and 13-10 for $\dot{m}_c C_{pc}$ and $\dot{m}_h C_{ph}$ and substituting into Eq. 13-23 gives, after some rearrangement,

$$\dot{Q} = UA_s\, \Delta T_{\text{lm}} \tag{13-24}$$

where

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{13-25}$$

is the **log mean temperature difference,** which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature difference between the two fluids

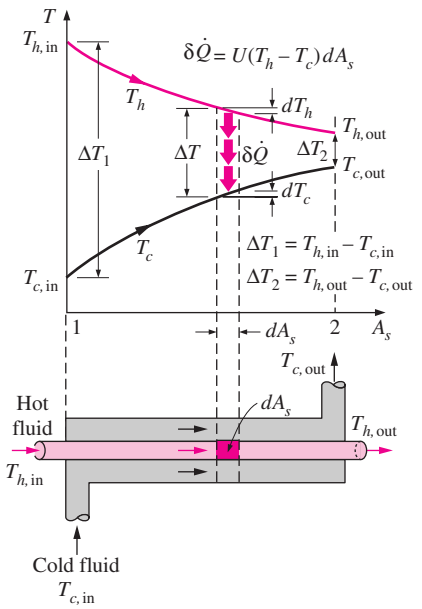
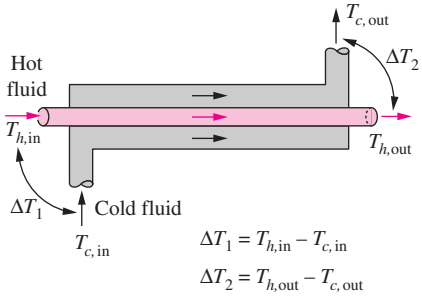
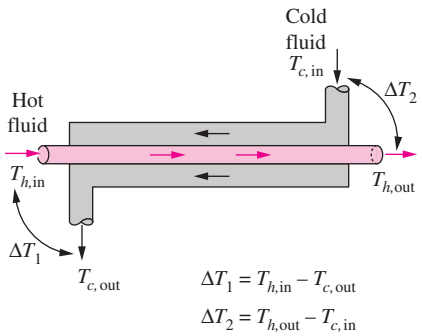


FIGURE 13–14
Variation of the fluid temperatures in a parallel-flow double-pipe heat exchanger.

cen58933_ch13.qxd 9/9/2002 9:57 AM Page 682



(*a*) Parallel-flow heat exchangers



(*b*) Counter-flow heat exchangers

FIGURE 13–15
The ΔT_1 and ΔT_2 expressions in parallel-flow and counter-flow heat exchangers.

at the two ends (inlet and outlet) of the heat exchanger. It makes no difference which end of the heat exchanger is designated as the *inlet* or the *outlet* (Fig. 13–15).

The temperature difference between the two fluids decreases from ΔT_1 at the inlet to ΔT_2 at the outlet. Thus, it is tempting to use the arithmetic mean temperature $\Delta T_{\text{am}} = \frac{1}{2}(\Delta T_1 + \Delta T_2)$ as the average temperature difference. The logarithmic mean temperature difference ΔT_{lm} is obtained by tracing the actual temperature profile of the fluids along the heat exchanger and is an *exact* representation of the *average temperature difference* between the hot and cold fluids. It truly reflects the exponential decay of the local temperature difference.

Note that ΔT_{lm} is always less than ΔT_{am}. Therefore, using ΔT_{am} in calculations instead of ΔT_{lm} will overestimate the rate of heat transfer in a heat exchanger between the two fluids. When ΔT_1 differs from ΔT_2 by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_1 differs from ΔT_2 by greater amounts. Therefore, we should always use the *logarithmic mean temperature difference* when determining the rate of heat transfer in a heat exchanger.

Counter-Flow Heat Exchangers

The variation of temperatures of hot and cold fluids in a counter-flow heat exchanger is given in Figure 13–16. Note that the hot and cold fluids enter the heat exchanger from opposite ends, and the outlet temperature of the *cold fluid* in this case may exceed the outlet temperature of the *hot fluid.* In the limiting case, the cold fluid will be heated to the inlet temperature of the hot fluid. However, the outlet temperature of the cold fluid can *never* exceed the inlet temperature of the hot fluid, since this would be a violation of the second law of thermodynamics.

The relation above for the log mean temperature difference is developed using a parallel-flow heat exchanger, but we can show by repeating the analysis above for a counter-flow heat exchanger that is also applicable to counter-flow heat exchangers. But this time, ΔT_1 and ΔT_2 are expressed as shown in Figure 13–15.

For specified inlet and outlet temperatures, the log mean temperature difference for a *counter-flow* heat exchanger is always *greater* than that for a parallel-flow heat exchanger. That is, $\Delta T_{\text{lm, }CF} > \Delta T_{\text{lm, }PF}$, and thus a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in a counter-flow heat exchanger. Therefore, it is common practice to use counter-flow arrangements in heat exchangers.

In a counter-flow heat exchanger, the temperature difference between the hot and the cold fluids will remain constant along the heat exchanger when the *heat capacity rates* of the two fluids are *equal* (that is, ΔT = constant when $C_h = C_c$ or $\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$). Then we have $\Delta T_1 = \Delta T_2$, and the last log mean temperature difference relation gives $\Delta T_{\text{lm}} = \frac{0}{0}$, which is indeterminate. It can be shown by the application of l'Hôpital's rule that in this case we have $\Delta T_{\text{lm}} = \Delta T_1 = \Delta T_2$, as expected.

A *condenser* or a *boiler* can be considered to be either a parallel- or counter-flow heat exchanger since both approaches give the same result.

cen58933_ch13.qxd 9/9/2002 9:57 AM Page 683

Multipass and Cross-Flow Heat Exchangers: Use of a Correction Factor

The log mean temperature difference ΔT_{lm} relation developed earlier is limited to parallel-flow and counter-flow heat exchangers only. Similar relations are also developed for *cross-flow* and *multipass shell-and-tube* heat exchangers, but the resulting expressions are too complicated because of the complex flow conditions.

In such cases, it is convenient to relate the equivalent temperature difference to the log mean temperature difference relation for the counter-flow case as

$$\Delta T_{\text{lm}} = F\,\Delta T_{\text{lm},\,CF} \tag{13-26}$$

where F is the **correction factor,** which depends on the *geometry* of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams. The $\Delta T_{\text{lm},\,CF}$ is the log mean temperature difference for the case of a *counter-flow* heat exchanger with the same inlet and outlet temperatures and is determined from Eq. 13-25 by taking $\Delta T_1 = T_{h,\,\text{in}} - T_{c,\,\text{out}}$ and $\Delta T_2 = T_{h,\,\text{out}} - T_{c,\,\text{in}}$ (Fig. 13–17).

The correction factor is less than unity for a cross-flow and multipass shell-and-tube heat exchanger. That is, $F \le 1$. The limiting value of $F = 1$ corresponds to the counter-flow heat exchanger. Thus, the correction factor F for a heat exchanger is *a measure of deviation of the* ΔT_{lm} *from the corresponding values for the counter-flow case.*

The correction factor F for common cross-flow and shell-and-tube heat exchanger configurations is given in Figure 13–18 versus two temperature ratios P and R defined as

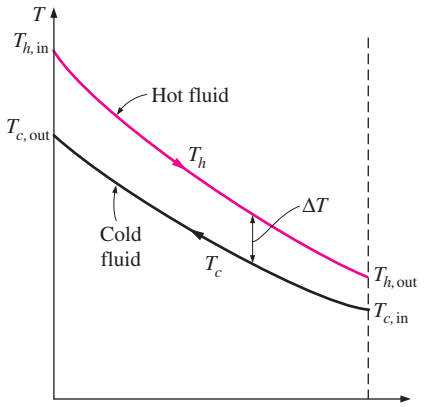
$$P = \frac{t_2 - t_1}{T_1 - t_1} \tag{13-27}$$

and

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}C_p)_{\text{tube side}}}{(\dot{m}C_p)_{\text{shell side}}} \tag{13-28}$$

where the subscripts 1 and 2 represent the *inlet* and *outlet,* respectively. Note that for a shell-and-tube heat exchanger, T and t represent the *shell-* and *tube-side* temperatures, respectively, as shown in the correction factor charts. It makes no difference whether the hot or the cold fluid flows through the shell or the tube. The determination of the correction factor F requires the availability of the *inlet* and the *outlet* temperatures for both the cold and hot fluids.

Note that the value of P ranges from 0 to 1. The value of R, on the other hand, ranges from 0 to infinity, with $R = 0$ corresponding to the phase-change (condensation or boiling) on the shell-side and $R \to \infty$ to phase-change on the tube side. The correction factor is $F = 1$ for both of these limiting cases. Therefore, the correction factor for a *condenser* or *boiler* is $F = 1$, regardless of the configuration of the heat exchanger.



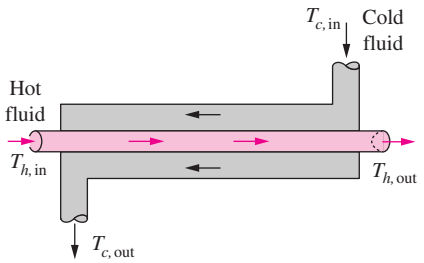
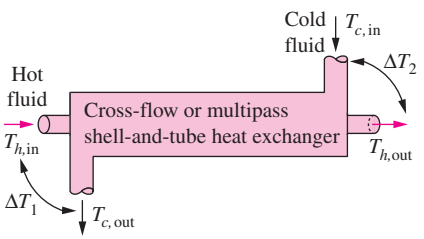


FIGURE 13–16
The variation of the fluid temperatures in a counter-flow double-pipe heat exchanger.



Heat transfer rate:

$$\dot{Q} = UA_s F \Delta T_{lm,CF}$$

where

$$\Delta T_{\text{lm},CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

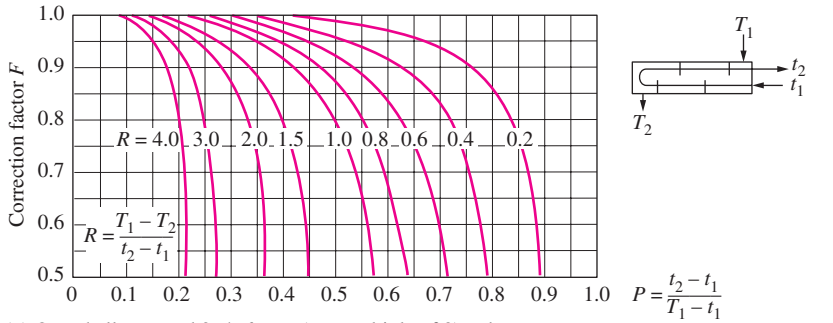
$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$$

$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$$

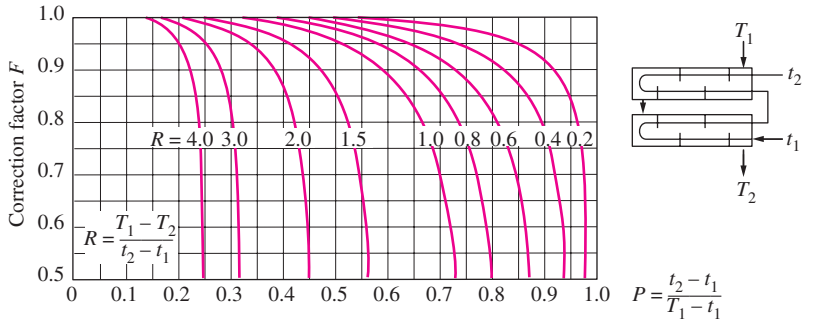
and F = … (Fig. 13–18)

FIGURE 13–17
The determination of the heat transfer rate for cross-flow and multipass shell-and-tube heat exchangers using the correction factor.

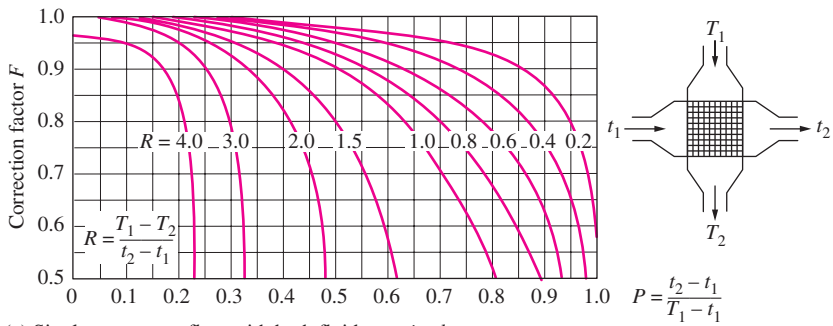
cen58933_ch13.qxd 9/9/2002 9:57 AM Page 684



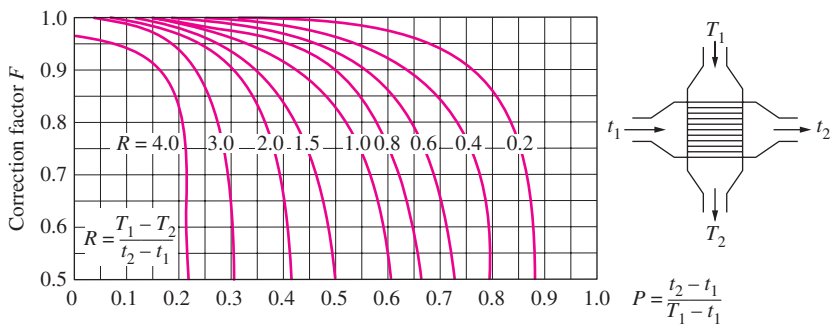
(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

FIGURE 13–18
Correction factor F charts for common shell-and-tube and cross-flow heat exchangers (from Bowman, Mueller, and Nagle, Ref. 2).

cen58933_ch13.qxd 9/9/2002 9:57 AM Page 685

EXAMPLE 13–3 The Condensation of Steam in a Condenser

Steam in the condenser of a power plant is to be condensed at a temperature of 30°C with cooling water from a nearby lake, which enters the tubes of the condenser at 14°C and leaves at 22°C. The surface area of the tubes is 45 m^2, and the overall heat transfer coefficient is 2100 $W/m^2 \cdot °C$. Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser.

SOLUTION Steam is condensed by cooling water in the condenser of a power plant. The mass flow rate of the cooling water and the rate of condensation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The heat of vaporization of water at 30°C is $h_{fg} = 2431$ kJ/kg and the specific heat of cold water at the average temperature of 18°C is $C_p = 4184$ J/kg · °C (Table A–9).

Analysis The schematic of the condenser is given in Figure 13–19. The condenser can be treated as a counter-flow heat exchanger since the temperature of one of the fluids (the steam) remains constant.

The temperature difference between the steam and the cooling water at the two ends of the condenser is

$$\Delta T_1 = T_{h,\,\text{in}} - T_{c,\,\text{out}} = (30 - 22)°\text{C} = 8°\text{C}$$
$$\Delta T_2 = T_{h,\,\text{out}} - T_{c,\,\text{in}} = (30 - 14)°\text{C} = 16°\text{C}$$

That is, the temperature difference between the two fluids varies from 8°C at one end to 16°C at the other. The proper average temperature difference between the two fluids is the *logarithmic mean temperature difference* (not the arithmetic), which is determined from

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8 - 16}{\ln(8/16)} = 11.5°\text{C}$$

This is a little less than the arithmetic mean temperature difference of $\frac{1}{2}(8 + 16) = 12°\text{C}$. Then the heat transfer rate in the condenser is determined from

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}} = (2100\ \text{W/m}^2 \cdot °\text{C})(45\ \text{m}^2)(11.5°\text{C}) = \mathbf{1.087 \times 10^6\ W = 1087\ kW}$$

Therefore, the steam will lose heat at a rate of 1,087 kW as it flows through the condenser, and the cooling water will gain practically all of it, since the condenser is well insulated.

The mass flow rate of the cooling water and the rate of the condensation of the steam are determined from $\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{cooling water}} = (\dot{m}h_{fg})_{\text{steam}}$ to be

$$\dot{m}_{\text{cooling water}} = \frac{\dot{Q}}{C_p(T_{\text{out}} - T_{\text{in}})}$$
$$= \frac{1{,}087\ \text{kJ/s}}{(4.184\ \text{kJ/kg} \cdot °\text{C})(22 - 14)°\text{C}} = \mathbf{32.5\ kg/s}$$

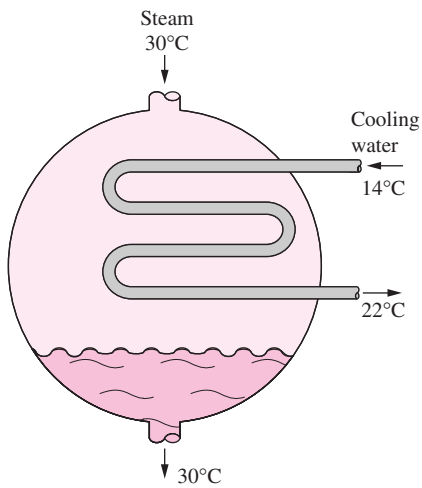


FIGURE 13–19
Schematic for Example 13–3.

cen58933_ch13.qxd 9/9/2002 9:57 AM Page 686

and

$$\dot{m}_{\text{steam}} = \frac{\dot{Q}}{h_{fg}} = \frac{1{,}087 \text{ kJ/s}}{2431 \text{ kJ/kg}} = \mathbf{0.45 \text{ kg/s}}$$

Therefore, we need to circulate about 72 kg of cooling water for each 1 kg of steam condensing to remove the heat released during the condensation process.

EXAMPLE 13–4 Heating Water in a Counter-Flow Heat Exchanger

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m^2 · °C, determine the length of the heat exchanger required to achieve the desired heating.

SOLUTION Water is heated in a counter-flow double-pipe heat exchanger by geothermal water. The required length of the heat exchanger is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties We take the specific heats of water and geothermal fluid to be 4.18 and 4.31 kJ/kg · °C, respectively.

Analysis The schematic of the heat exchanger is given in Figure 13–20. The rate of heat transfer in the heat exchanger can be determined from

$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot \text{°C})(80 - 20)\text{°C} = 301 \text{ kW}$$

Noting that all of this heat is supplied by the geothermal water, the outlet temperature of the geothermal water is determined to be

$$\dot{Q} = [\dot{m}C_p(T_{\text{in}} - T_{\text{out}})]_{\text{geothermal}} \longrightarrow T_{\text{out}} = T_{\text{in}} - \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 160\text{°C} - \frac{301 \text{ kW}}{(2 \text{ kg/s})(4.31 \text{ kJ/kg} \cdot \text{°C})}$$

$$= 125\text{°C}$$

Knowing the inlet and outlet temperatures of both fluids, the logarithmic mean temperature difference for this counter-flow heat exchanger becomes

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}} = (160 - 80)\text{°C} = 80\text{°C}$$
$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}} = (125 - 20)\text{°C} = 105\text{°C}$$

and

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{80 - 105}{\ln(80/105)} = 92.0\text{°C}$$

Then the surface area of the heat exchanger is determined to be

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}} \longrightarrow A_s = \frac{\dot{Q}}{U\,\Delta T_{\text{lm}}} = \frac{301{,}000 \text{ W}}{(640 \text{ W/m}^2 \cdot \text{°C})(92.0\text{°C})} = 5.11 \text{ m}^2$$

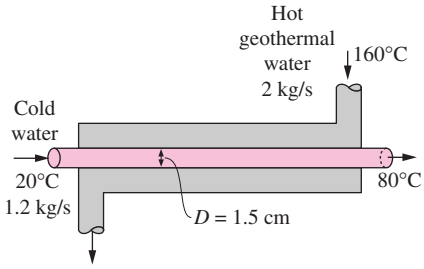


FIGURE 13–20
Schematic for Example 13–4.

cen58933_ch13.qxd 9/9/2002 9:57 AM Page 687

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11\ \text{m}^2}{\pi(0.015\ \text{m})} = \mathbf{108\ m}$$

Discussion The inner tube of this counter-flow heat exchanger (and thus the heat exchanger itself) needs to be over 100 m long to achieve the desired heat transfer, which is impractical. In cases like this, we need to use a plate heat exchanger or a multipass shell-and-tube heat exchanger with multiple passes of tube bundles.

EXAMPLE 13–5 Heating of Glycerin in a Multipass Heat Exchanger

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C (Fig. 13–21). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m^2 · °C on the glycerin (shell) side and 160 W/m^2 · °C on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (*a*) before any fouling occurs and (*b*) after fouling with a fouling factor of 0.0006 m^2 · °C/W occurs on the outer surfaces of the tubes.

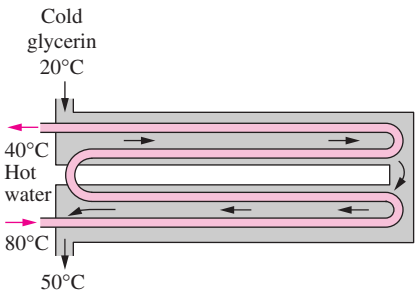


FIGURE 13–21
Schematic for Example 13–5.

SOLUTION Glycerin is heated in a 2-shell passes and 4-tube passes heat exchanger by hot water. The rate of heat transfer for the cases of fouling and no fouling are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Analysis The tubes are said to be thin-walled, and thus it is reasonable to assume the inner and outer surface areas of the tubes to be equal. Then the heat transfer surface area becomes

$$A_s = \pi DL = \pi(0.02\ \text{m})(60\ \text{m}) = 3.77\ \text{m}^2$$

The rate of heat transfer in this heat exchanger can be determined from

$$\dot{Q} = UA_s F\,\Delta T_{\text{lm},\,CF}$$

where F is the correction factor and $\Delta T_{\text{lm},\,CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are determined from

$$\Delta T_1 = T_{h,\,\text{in}} - T_{c,\,\text{out}} = (80 - 50)°\text{C} = 30°\text{C}$$
$$\Delta T_2 = T_{h,\,\text{out}} - T_{c,\,\text{in}} = (40 - 20)°\text{C} = 20°\text{C}$$
$$\Delta T_{\text{lm},\,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{30 - 20}{\ln(30/20)} = 24.7°\text{C}$$

cen58933_ch13.qxd 9/9/2002 9:57 AM Page 688

and

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 80}{20 - 80} = 0.67$$
$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75$$
$$\Big\} F = 0.91 \qquad \text{(Fig. 13–18}b\text{)}$$

(*a*) In the case of no fouling, the overall heat transfer coefficient U is determined from

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{160\ \text{W/m}^2 \cdot \text{°C}} + \frac{1}{25\ \text{W/m}^2 \cdot \text{°C}}} = 21.6\ \text{W/m}^2 \cdot \text{°C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UA_s F\,\Delta T_{\text{lm, }CF} = (21.6\ \text{W/m}^2 \cdot \text{°C})(3.77\text{m}^2)(0.91)(24.7\text{°C}) = \mathbf{1830\ W}$$

(*b*) When there is fouling on one of the surfaces, the overall heat transfer coefficient U is

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o} + R_f} = \frac{1}{\frac{1}{160\ \text{W/m}^2 \cdot \text{°C}} + \frac{1}{25\ \text{W/m}^2 \cdot \text{°C}} + 0.0006\ \text{m}^2 \cdot \text{°C/W}}$$
$$= 21.3\ \text{W/m}^2 \cdot \text{°C}$$

The rate of heat transfer in this case becomes

$$\dot{Q} = UA_s F\,\Delta T_{\text{lm, }CF} = (21.3\ \text{W/m}^2 \cdot \text{°C})(3.77\ \text{m}^2)(0.91)(24.7\text{°C}) = \mathbf{1805\ W}$$

Discussion Note that the rate of heat transfer decreases as a result of fouling, as expected. The decrease is not dramatic, however, because of the relatively low convection heat transfer coefficients involved.

EXAMPLE 13–6 Cooling of an Automotive Radiator

A test is conducted to determine the overall heat transfer coefficient in an automotive radiator that is a compact cross-flow water-to-air heat exchanger with both fluids (air and water) unmixed (Fig. 13–22). The radiator has 40 tubes of internal diameter 0.5 cm and length 65 cm in a closely spaced plate-finned matrix. Hot water enters the tubes at 90°C at a rate of 0.6 kg/s and leaves at 65°C. Air flows across the radiator through the interfin spaces and is heated from 20°C to 40°C. Determine the overall heat transfer coefficient U_i of this radiator based on the inner surface area of the tubes.

SOLUTION During an experiment involving an automotive radiator, the inlet and exit temperatures of water and air and the mass flow rate of water are measured. The overall heat transfer coefficient based on the inner surface area is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Changes in the kinetic and potential energies of fluid streams are negligible. **3** Fluid properties are constant.

cen58933_ch13.qxd 9/9/2002 9:57 AM Page 689

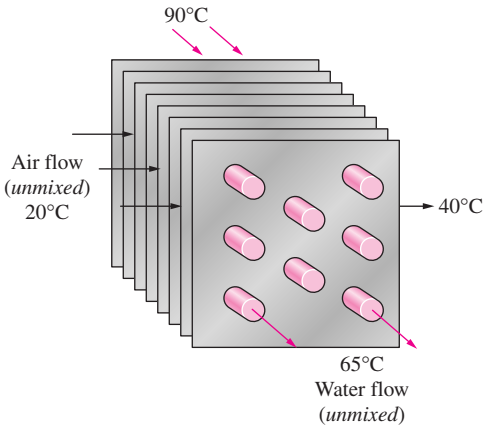


FIGURE 13–22
Schematic for Example 13–6.

Properties The specific heat of water at the average temperature of (90 + 65)/2 = 77.5°C is 4.195 kJ/kg · °C.

Analysis The rate of heat transfer in this radiator from the hot water to the air is determined from an energy balance on water flow,

$$\dot{Q} = [\dot{m}C_p(T_{\text{in}} - T_{\text{out}})]_{\text{water}} = (0.6\ \text{kg/s})(4.195\ \text{kJ/kg} \cdot °\text{C})(90 - 65)°\text{C} = 62.93\ \text{kW}$$

The tube-side heat transfer area is the total surface area of the tubes, and is determined from

$$A_i = n\pi D_i L = (40)\pi(0.005\ \text{m})(0.65\ \text{m}) = 0.408\ \text{m}^2$$

Knowing the rate of heat transfer and the surface area, the overall heat transfer coefficient can be determined from

$$\dot{Q} = U_i A_i F\, \Delta T_{\text{lm},\,CF} \quad \longrightarrow \quad U_i = \frac{\dot{Q}}{A_i F\, \Delta T_{\text{lm},\,CF}}$$

where F is the correction factor and $\Delta T_{\text{lm},\,CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are found to be

$$\Delta T_1 = T_{h,\,\text{in}} - T_{c,\,\text{out}} = (90 - 40)°\text{C} = 50°\text{C}$$
$$\Delta T_2 = T_{h,\,\text{out}} - T_{c,\,\text{in}} = (65 - 20)°\text{C} = 45°\text{C}$$
$$\Delta T_{\text{lm},\,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{50 - 45}{\ln(50/45)} = 47.6°\text{C}$$

and

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{65 - 90}{20 - 90} = 0.36 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 40}{65 - 90} = 0.80 \end{aligned}\right\} F = 0.97 \qquad \text{(Fig. 13–18}c\text{)}$$

Substituting, the overall heat transfer coefficient U_i is determined to be

$$U_i = \frac{\dot{Q}}{A_i F\, \Delta T_{\text{lm},\,CF}} = \frac{62{,}930\ \text{W}}{(0.408\ \text{m}^2)(0.97)(47.6°\text{C})} = \mathbf{3341\ W/m^2 \cdot °C}$$

Note that the overall heat transfer coefficient on the air side will be much lower because of the large surface area involved on that side.